HW 6 Solutions

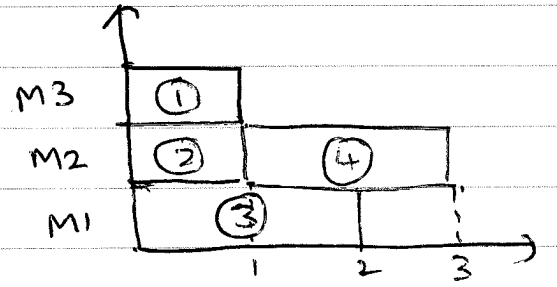
1.

a) Min C

Subj to
$$2x_{11} + 2x_{12} + 2x_{13} + 4x_{14} - C \leq 0$$
$$3x_{21} + x_{22} + 2x_{23} + 2x_{24} - C \leq 0$$
$$x_{31} + 3x_{32} + 4x_{33} + 4x_{34} - C \leq 0$$
$$x_{11} + x_{21} + x_{31} = 1$$
$$x_{12} + x_{22} + x_{32} = 1$$
$$x_{13} + x_{23} + x_{33} = 1$$
$$x_{14} + x_{24} + x_{34} = 1$$
All $x_{ij} \in \{0, 1\}$

b) $x_{13} = 1$, $x_{14} = 0.0625$, $x_{22} = 1$, $x_{24} = 0.625$
$x_{31} = 1$, $x_{34} = 0.3125$, $C = 2.25$
Using QSopt

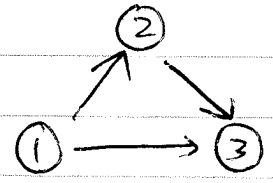
c) Integral solution:
$C_{max} = 3$



Why optimal?
- Since all p_{ij}'s are integral, C_{max} will be integral.
- Since LP > 2, $C_{max} > 2$
- $\therefore C_{max} = 3$ is optimal

$$\text{Integrality Gap} = \frac{3}{2.25} = \frac{4}{3}$$

2. $P_1 = 3,\ P_2 = 4,\ P_3 = 2$



$p(\{1,2\}) = 7 \qquad p^2(\{1,2\}) = 25$
$p(\{1,3\}) = 5 \qquad p^2(\{1,3\}) = 13$
$p(\{2,3\}) = 6 \qquad p^2(\{2,3\}) = 20$
$p(\{1,2,3\}) = 9 \qquad p^2(\{1,2,3\}) = 29$
$p(\{1\}) = 3 \qquad p^2(\{1\}) = 9$
$p(\{2\}) = 4 \qquad p^2(\{2\}) = 16$
$p(\{3\}) = 2 \qquad p^2(\{3\}) = 4$

$\underline{P}$:

minimize $5C_1 + 12C_2 + 7C_3$

subject to:

$$C_2 \geq C_1 + 4$$
$$C_3 \geq C_1 + 2$$
$$C_3 \geq C_2 + 2$$

$$\begin{aligned}
3C_1 + 4C_2 &\geq 37 \\
3C_1 + 2C_3 &\geq 19 \\
4C_2 + 2C_3 &\geq 28 \\
3C_1 + 4C_2 + 2C_3 &\geq 55 \\
C_1 &\geq 3 \\
C_2 &\geq 4 \\
C_3 &\geq 2
\end{aligned}$$

Solution: $(1) \to (2) \to (3)$ $\qquad \sum w_j C_j = 162$

$C_1 = 3,\ C_2 = 7,\ C_3 = 9$

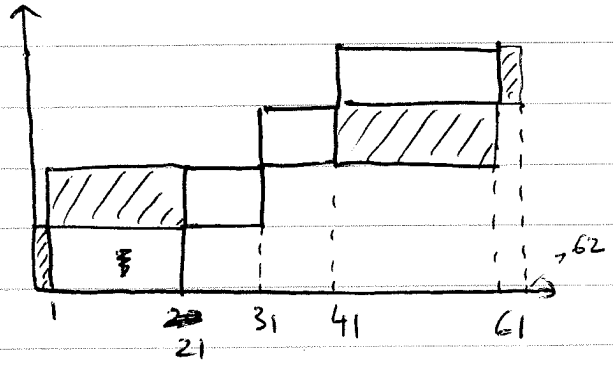
3. Consider the following instance

M/c	Job 1	Job 2
1	20	1
2	10	20
3	10	20
4	20	1

Optimal Schedule

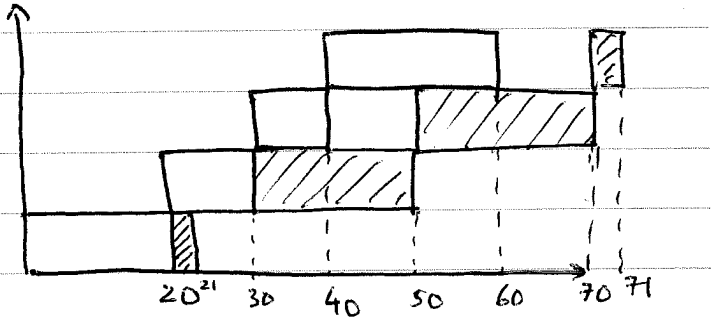
Job 1 ▭

Job 2 ▭

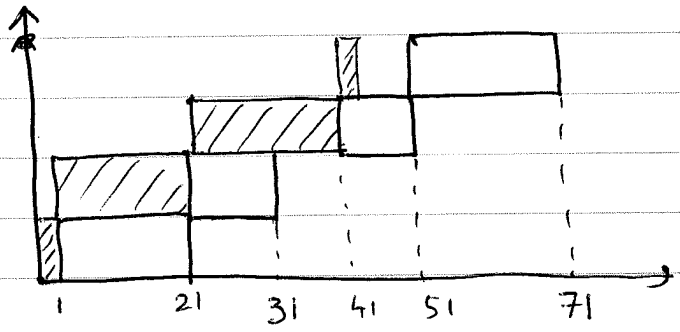


Note this is not permutation

If perm. is $\{1,2\}$:



If perm is $\{2,1\}$

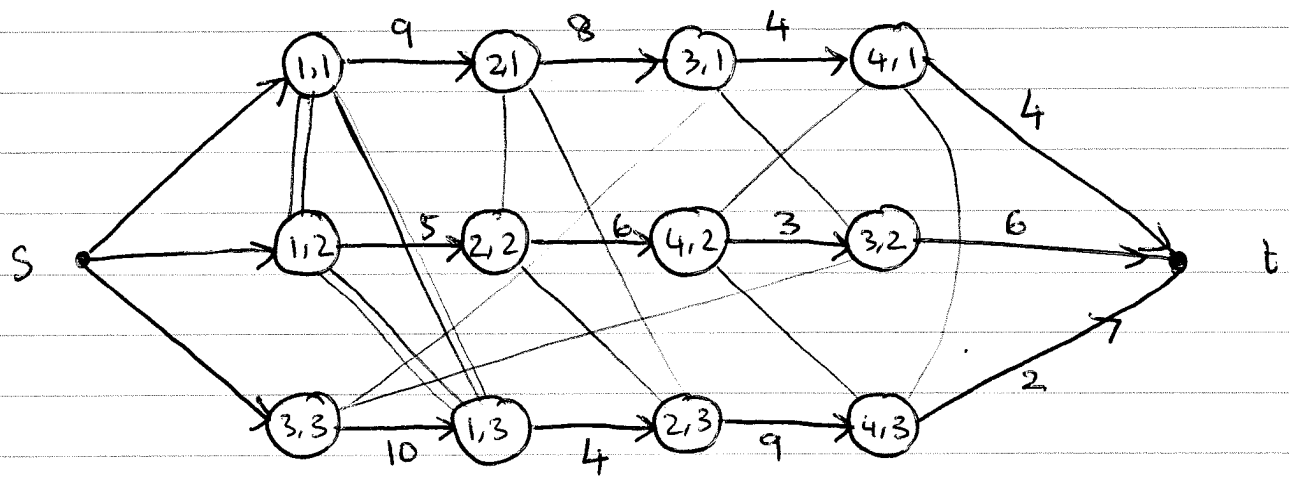


$\therefore$ $OPT(F4|prmu|C_{max}) = 71$

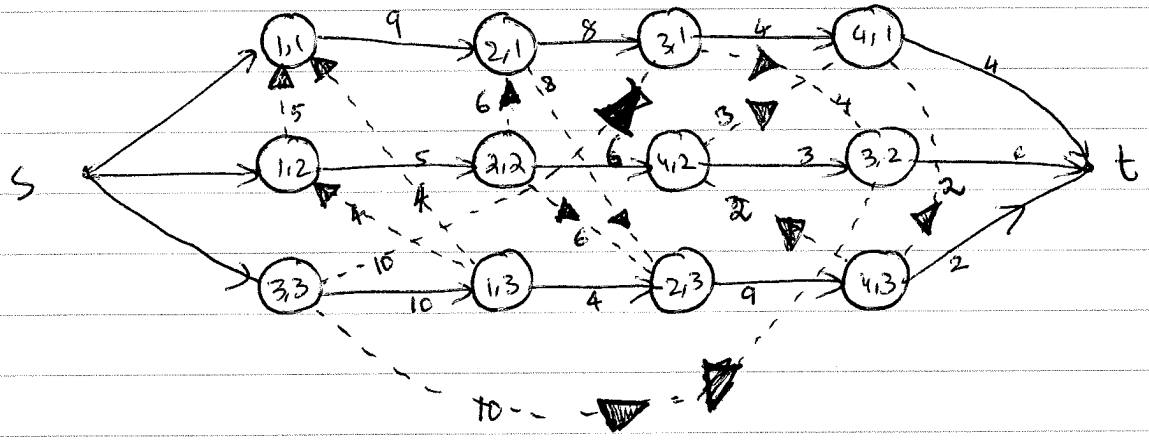
$OPT(F4||C_{max}) = 62$

(4)

a)



b)

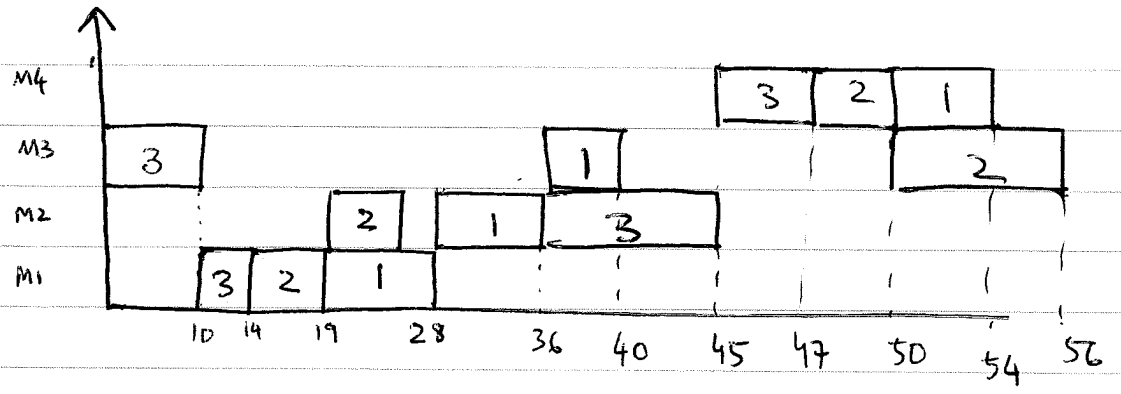


$t_{3,3} = 0$, $t_{1,3} = 10$, $t_{1,2} = 14$, $t_{1,1} = 19$

$t_{2,2} = 19$, $t_{2,1} = 28$, $t_{3,1} = 36$, $t_{2,3} = 36$

$t_{4,3} = 45$, $t_{4,2} = 47$, $t_{4,1} = 50$, $t_{3,2} = 50$

$\therefore C_{max} = 56$



Schedule from preceding orientation

$M = 70$ suffices

c) Minimize C

$C \geq C_{41}$, $C \geq C_{32}$, $C \geq C_{43}$

$C_{21} \geq C_{11} + 8$	$C_{22} \geq C_{12} + 6$	$C_{13} \geq C_{33} + 4$
$C_{31} \geq C_{21} + 4$	$C_{42} \geq C_{22} + 3$	$C_{23} \geq C_{13} + 9$
$C_{41} \geq C_{31} + 4$	$C_{32} \geq C_{42} + 6$	$C_{43} \geq C_{23} + 2$
$C_{11} \geq 9$	$C_{12} \geq 5$	$C_{33} \geq 10$

$M x_{13,11} + C_{13} \geq C_{11} + 4$	$M x_{12,13} + C_{12} \geq C_{13} + 5$	$M x_{11,12} + C_{11} \geq C_{12} + 9$
$M x_{11,13} + C_{11} \geq C_{13} + 9$	$M x_{13,12} + C_{13} \geq C_{12} + 4$	$M x_{12,11} + C_{12} \geq C_{11} + 5$

$M x_{21,22} + C_{21} \geq C_{22} + 8$	$M x_{21,23} + C_{21} \geq C_{23} + 8$	$M x_{22,23} + C_{22} \geq C_{23} + 6$
$M x_{22,21} + C_{22} \geq C_{21} + 6$	$M x_{23,21} + C_{23} \geq C_{21} + 9$	$M x_{23,22} + C_{23} \geq C_{22} + 9$

$M x_{31,32} + C_{31} \geq C_{32} + 4$	$M x_{31,33} + C_{31} \geq C_{33} + 4$	$M x_{32,33} + C_{32} \geq C_{33} + 6$
$M x_{32,31} + C_{32} \geq C_{31} + 6$	$M x_{33,31} + C_{33} \geq C_{31} + 10$	$M x_{33,32} + C_{33} \geq C_{32} + 10$

$M x_{41,42} + C_{41} \geq C_{42} + 4$	$M x_{41,43} + C_{41} \geq C_{43} + 4$	$M x_{42,43} + C_{42} \geq C_{43} + 3$
$M x_{42,41} + C_{42} \geq C_{41} + 3$	$M x_{43,41} + C_{43} \geq C_{41} + 2$	$M x_{43,42} + C_{43} \geq C_{42} + 2$

$X_{13,11} + X_{11,13} = 1$, $X_{12,13} + X_{13,12} = 1$, $X_{11,12} + X_{12,11} = 1$

$\forall i \in S, i \neq$