Approximation Algorithms

EO249

(Problem Set 1) Due: Feb 6th, 2015

Solutions need to be submitted by email to eo249iisc@gmail.com. We prefer latexed solution. Before giving the solutions, you should write how many problems you have attempted and how many you think you have solved. Starred problems are optional and (possibly) more fun.

Exercise 1. Write a **formal** proof to establish the following matching lower bound for TSP: For any subset O of even cardinality, show that the cost of the minimum cost matching M_O satisfies $c(M_O) \leq \frac{1}{2}OPT_{TSP}$.

Exercise 2. Prove that it is NP-hard to obtain a polynomial time α-approximation for any α for the general TSP. Hint: Use the fact that Hamiltonian Cycle problem is NP-complete.

Exercise 3. Give an example to show that the analysis of the 1.5-approximation for TSP done in class is almost tight. In particular, come up with an instance of $n$points and metric costs, an MST T, a minimum cost matching M_O where O is the set of odd vertex nodes of T, such that $c(T + M_O) \geq (1.5 - g(n))OPT$ for some function g which goes to 0 as n goes to infinity.

Exercise 4. Given a connected multigraph G where every degree is even, give a polynomial time algorithm to find an Eulerian walk. Given a strongly connected multigraph G where every vertex has in-degree equal to out-degree, give a polynomial time algorithm to find an Eulerian walk.

Exercise 5. Design a polynomial time algorithm to find the minimum cost cycle cover of a directed graph G. Hint: Consider the bipartite graph $H = (A, B)$ where A and B are copies of vertices of G and there is an edge from $u \in A$ to $v \in B$ iff there is a directed edge from u to v in G. What do cycle covers in G correspond to in H?

Exercise 6. (*) The Travelling salesman path problem, or simply TSPP is the following: given n points $\{1, \ldots, n\}$ and symmetric costs $c(i, j) = c(j, i)$ satisfying the metric property $c(i, k) \leq c(i, j) + c(j, k)$ for all i, j, k, find the minimum cost path starting from 1 and ending at n and traversing all other vertices once.

1. Show that TSPP is NP-hard to solve.
2. What are some lower bounds on the optimum TSPP? Is MST a lower bound?
3. What is the best factor approximation algorithm you can find for TSPP?