What should I know before taking CS 30

- **Boolean Variables.** Boolean variables or simply booleans are the most basic unit of data: the *bit*. The bit takes two values: 1 or "True", and 0 or "False".

- **Numbers and Arithmetic.** There are many types of numbers: *natural/whole numbers* $\{0, 1, 2, 3, \ldots, \}$, *integers* $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$, which also include negative numbers, *rational numbers*, which are of the form p/q where both p and q are integers, and *real* numbers which can represented on the number line can be approximately represented by *decimals*.

- **Intervals on the Number Line.** For any two reals, $a \leq b$, we define the interval $[a, b]$ to be all reals x such that $a \leq x \leq b$. If any of the square brackets is replaced by a paranthesis, then the corresponding inequality becomes a strict equality. So, $(a, b]$ denotes all x such that $a < x \leq b$.

- **Arithmetic.** Any two numbers can be added, subtracted, multiplied, and divided. Rationals are *closed* under these *operations*, that is, the sum/difference/product/ratio of any two rationals is rational. This is *not true* for integers.

- **Prime and Composite Numbers.** A positive number p is *prime* if the only numbers dividing it (that is, leaving 0 remainder) are 1 and p. Otherwise, the number is composite. The only exception to the rule is 1 which is neither prime nor composite. For instance, 17 is a prime, but $323 = 17 \times 19$ is not.

- **GCD and Relatively Prime Numbers.** The *greatest common divisor* (GCD) of two positive numbers a and b is the largest number g which divides both a and b. So the GCD of 15 and 24 is 3. Two numbers are *coprime* or *relatively prime* if their GCD is 1. Two *different* prime numbers are clearly coprime (do you see why?), for instance $(3, 5)$ are coprime. But neither of these numbers need to be primes; for instance $(10, 21)$ are coprime but neither are primes.

- **Prime Factorization.** Any composite number n can be *uniquely* written as a product of primes (uniquely upto changing the ordering of multiplication). This is the *prime factorization theorem*. This is a non-trivial fact which we may or may not prove in CS 30.

- **Floors and Ceilings.** Given any real number x, the *floor* $\lfloor x \rfloor$ is the *largest integer smaller than or equal to* x. So, $\lfloor 2 \rfloor = 2$, and $\lfloor 1.5 \rfloor = 1$, and $\lfloor -2.7 \rfloor = -3$. Similarly, the *ceiling* $\lceil x \rceil$ is the *smallest integer larger than or equal to* x. So, $\lceil 2 \rceil = 2$ and $\lceil 1.5 \rceil = 2$ and $\lceil -2.7 \rceil = -2$.

- **Exponentiating.** Given any positive integer n and any real x, the number x^n is a shorthand for $x \cdot x \cdot x \cdots x$, where x is multiplied with itself n times. So, $3^2 = 9$ and $(1.5)^2 = 2.25$ and $(-1)^3 = -1$.

  The number x^0 is defined to be 1 for *any* x. So, $3^0 = 1$ and $(-1)^0 = 1$, and *also* $0^0 = 1$. The last is really tricky – what is 0 multiplied by itself never ... well in multiplication you assume there is a "base" 1 to which things are being multiplied, and if 0 is multiplied to it never, we will have 1.

  For any positive rational number p/q, we define $x^{p/q}$ as the number y such that y^q (that is y multiplied by itself q times) equals x^p. If there are more than one such value, we will take the largest one. For example, $4^{1/2} = 2$ since $2^2 = 4$, although $(-2)^2 = 4$, as well. On the other hand, $(-27)^{1/3} = -3$. In general, y need not be an integer, nor a rational number, but if x is positive, it is always a *real number*. The special case of $p = 1$ is called the *qth root*; in particular, the 2th root of x is called the *square root*, and is denoted as $\sqrt{x}$.

For a positive real number y, the definition of x^y requires limits. We can approximate x^y by taking a rational number p/q which approximates y well, and then returning $x^{p/q}$.

Finally, for any negative number $y = -z$, we define $x^y = x^{-z} := 1/x^z$.

Some properties of exponentials: For any reals x, y and any rational a, b, we have

1. $x^{a+b} = x^a \cdot x^b$
2. $(xy)^a = x^a \cdot y^a$
3. $(x^a)^b = x^{ab}$

The above properties follow easily if a and b were integers. Do you see why this is true for rationals?

- **Logarithms.** The *logarithm* of a *positive* number a to the *positive base* $b \neq 1$, denoted as $\log_b a$ is the number ℓ such that $b^\ell = a$. Thus, $\log_3 81 = 4$ (since $3^4 = 81$) and $\log_{4/3}(16/9) = 2$ (since $(4/3)^2 = 16/9$), and $\log_{\sqrt{2}} 16 = 4$ (since $(\sqrt{2})^8 = 16$).

  Do you see why the logarithm of a non-negative number is undefined? If there were a number $\ell = \log_b a$, then $a = b^\ell$. But if b is positive, then b^ℓ *even when ℓ is negative* is always positive.

  Some properties of logarithms: These properties follow by using the properties of exponential and the definition of logarithms

  1. $\log_b 1 = 0$ for all $b > 0$.
  2. $\log_b(xy) = \log_b x + \log_b y$ for all $b, x, y > 0$.
  3. $\log_b(x^y) = y \log_b x$ for all $b, x, y > 0$.
  4. $\log_b x = \frac{\log_c x}{\log_c b}$.

  The above properties follow easily if a and b were integers. Do you see why this is true for rationals?

- **Basic Formulae.** You should be able to deduce the following

  1. For any two numbers, $(a + b)^2 = a^2 + 2ab + b^2$
  2. For any two numbers, $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
  3. For any three numbers, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

- **Calculus.** You should brush up your basic calculus a bit (we won't use it much). You should, for example, know the answers to what $\frac{dx^3}{dx}$ and $\frac{de^x}{dx}$ are, and what $\int_4^{19} x^2 dx$ is.

To figure out if you know all this stuff, try the *first 48* problems in Section 2.2.8 (Exercises) from the textbook (don't hand them in.).