

CS 30: Discrete Math in CS (Winter 2019): Lecture 2 Supplement

Date: 4th January, 2019 (Fri)

Topic: More examples of Proofs by Contradiction

Disclaimer: These notes have not gone through scrutiny and in all probability contain errors.

Please discuss in Piazza/email errors to deeparnab@dartmouth.edu

This material was not covered in class but supplements the stuff that was covered. Think of this material as what I would've covered if I had a semester instead of a quarter.

- **Logarithms in different bases.**

Theorem 1. $\log_2 3$ is irrational.

Proof.

1. For the sake of contradiction, assume $\log_2 3$ is rational.
2. Then there exists integers a and b such that $\log_2 3 = \frac{a}{b}$.
3. That is, $b \log_2 3 = a$.
4. Which implies $\log_2 (3^b) = a$.
5. That is, $3^b = 2^a$.
6. But this is a contradiction since the LHS is odd while the RHS is even.

□



Exercise: Is $\log_{\sqrt{2}} 3$ rational or irrational?



Exercise: Can you now figure out two irrational numbers x and y such that x^y is rational?